Game theory and applications: from opinion dynamics to electricity markets

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Motivation

- **Games**: strategic interactions (Nash (1950))
- **Coordinating agents**: spread of social norms and innovations
- **Anti-coordinating agents**: traffic congestion, crowd dispersion and division of labor
- Irregular network topology and population heterogeneity are not sufficient to cause nonexistence of Nash equilibria; **coexistence of coordinating and anti-coordinating agents** must play a role (Ramazi et al. (2016))

Model description

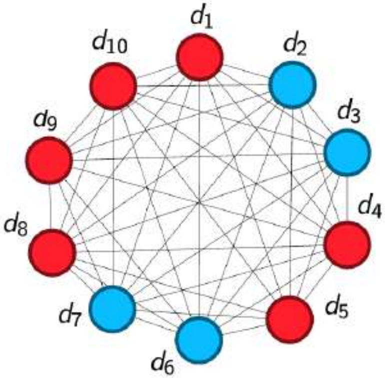
Mixed coordination anti-coordination (CAC) game

- Finite agent set $\mathcal{N} = \mathcal{N}_c \cup \mathcal{N}_a$
  - $\mathcal{N}_c$ coordinating agents
  - $\mathcal{N}_a$ anti-coordinating agents

⋆ Agent types $\delta_i = \begin{cases} 1 & \text{if } i \in \mathcal{N}_c \\ -1 & \text{if } i \in \mathcal{N}_a \end{cases}$

⋆ Agent weights $\{d_i\}_{i \in \mathcal{V}}$, $d_i \in \mathbb{R}$

- Binary action set $\mathcal{A} = \{-1, +1\}$

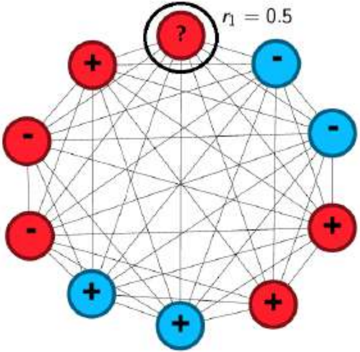


Utility $i \in \mathcal{N}$:
$$u_i(x_i, x_{-i}) = \delta_i \left(\sum_{j \in \mathcal{N}} x_i x_j - d_i x_i \right)$$

when action $x_i \in \mathcal{A}$ and actions of others $x_{-i} \in \mathcal{A}^{\mathcal{N} \setminus \{i\}}$.

Coordinating agent $i \in \mathcal{N}_c$

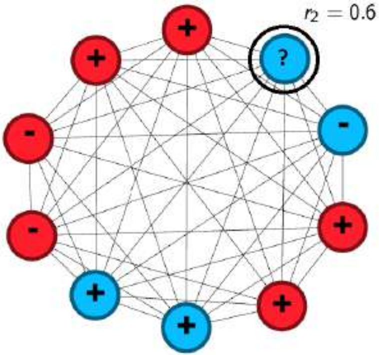
$$\mathcal{B}_i(x_{-i}) := \arg\max_{x_i \in \mathcal{A}} u_i(x_i, x_{-i})$$



- Thresholds: $r_i = \frac{1}{2} + \frac{d_i}{2(n-1)}$.

Anti-coordinating agent $i \in \mathcal{N}_a$

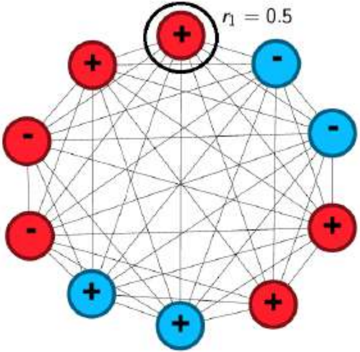
$$\mathcal{B}_i(x_{-i}) := \arg\max_{x_i \in \mathcal{A}} u_i(x_i, x_{-i})$$



Best response function

Coordinating agent $i \in \mathcal{N}_c$

$$\mathcal{B}_i(x_{-i}) := \arg\max_{x_i \in \mathcal{A}} u_i(x_i, x_{-i})$$



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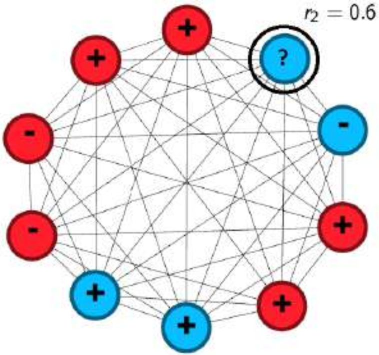

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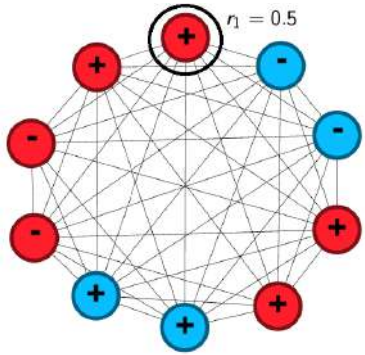

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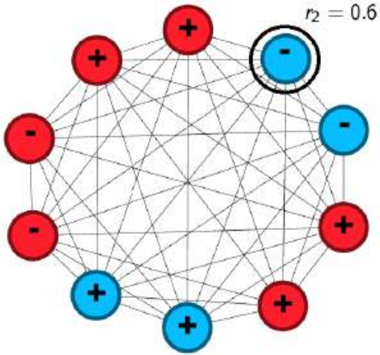


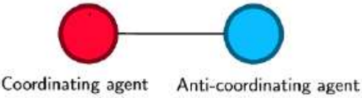
Goal

Investigate existence and characterization of Nash equilibria.

- **Nash equilibrium:** no incentive in unilaterally changing the action ($x_i^* \in \mathcal{B}_i(x_{-i}^*)$ for all i).
- **Potential game** (Monderer and Shapley (1996))
  $\rightarrow$ Existence of Nash equilibria guaranteed
- Both the coordination game ($\mathcal{N}_a = \emptyset$) and the anti-coordination game ($\mathcal{N}_c = \emptyset$) are potential games.

Proposition

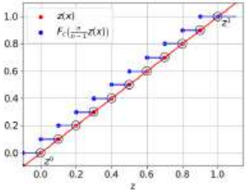
*One **interaction** between a **coordinating agent** and an **anti-coordinating agent** $\rightarrow$ **not** a potential game*



The discoordination game admits no Nash equilibria

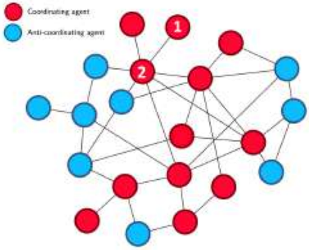
Main results and current work

- Checkable condition on thresholds for the existence of NE
- Straightforward algorithm to derive all NE
- Cardinality when only coordinating agents or only anti-coordinating agents are present.



Current work

- Extensions to agents confined to interact through a **network**.
  - → Sufficient condition on network topology for the *existence* of NE (cohesiveness of coordinating agents)
  - → Sufficient condition on network topology for *global reachability* of NE (undecomposability of coordinating agents)



- Extensions to **robustness** of **network coordination games** (perturbation of a potential game).

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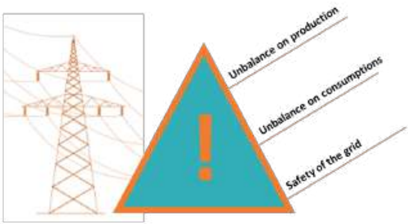
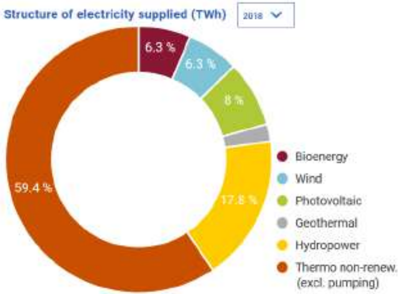
- (Progressive) liberalization of electricity markets.
- **Electricity** cannot be stored (if not in a negligible way)
- **Dispatching**: instant by instant management of the energy flows that pass through the transmission network
  $\rightarrow$ continuous balance between quantity introduced into the network and quantity withdrawn from it.

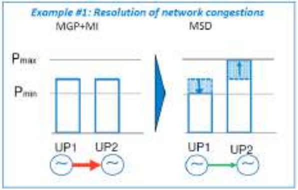


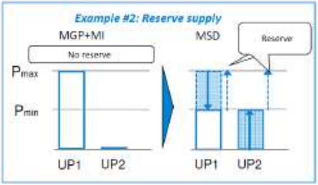


Source: www.terna.it

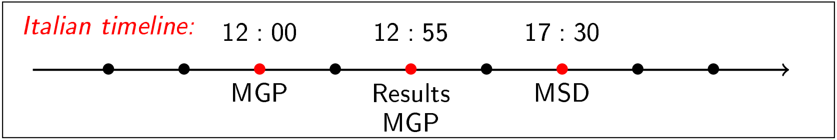
Electricity markets: hourly auctions

1. Day-Ahead Market
   - Most of the energy, **uniform price** (intersection of supply/offer curves).
   - ⋆ Mercato del Giorno Prima (MGP)
2. Ancillary Services Market
   - Lower volumes, **pay-as-bid**.
   - Resolution of congestion and reserve margin





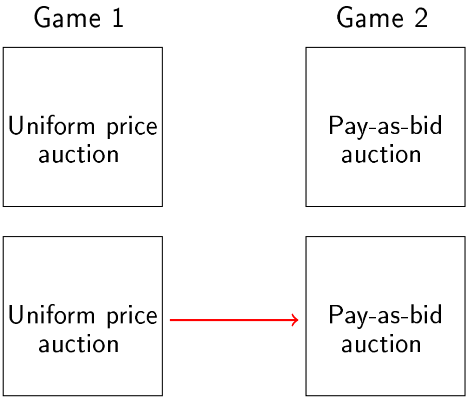
   - Real-time balancing
   - ⋆ Mercato dei Servizi di Dispacciamento (MSD)



Goal

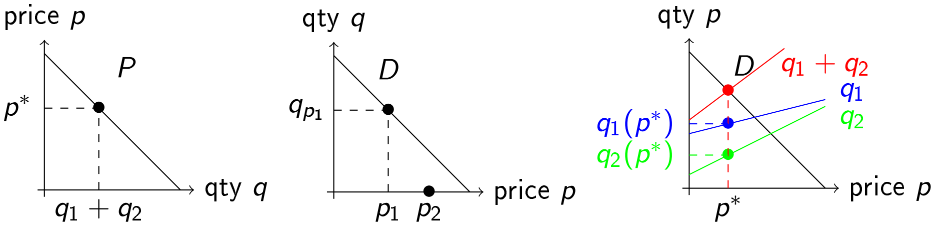
Study the **uniform-price auction** and the **pay-as-bid auction** first as separated games and then as a two-stage game.

- Existence and uniqueness of Nash equilibria.
- Mechanism design.



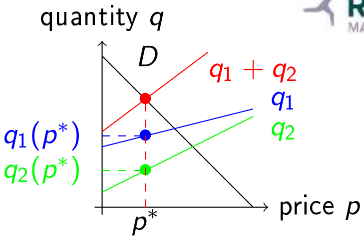
Classification of strategies according to the structure of the offers:

1. **Cournot competition**: each firm bids a quantity (1838).
2. **Bertrand competition**: each firm bids a price (1883).
3. **Supply Function Equilibria** (Klemperer and Meyer (1989)): each firm bids a function of the price.



Model description: uniform price auction

- 2 firms
- **Cost functions** $C(q)$, q quantity.
- **Demand function** $D(p)$, p price.



Definition (Uniform price auction)

1. Agent set $\mathcal{N} = \{1, 2\}$
2. Action set:

$$\mathcal{A} := \{q : \mathbb{R}^+ \to \mathbb{R}\,,\; q \text{ non-decreasing function of price}\}$$

3. Utility of firm $i \in \mathcal{N}$:

$$u_i^{\mathrm{U}}(q_i, q_j) = p^* q_i(p^*) - C(q_i(p^*))\,,$$

where p^* is the equilibrium price that equates total demand and total supply.

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- **Cost functions** $C(q)$, q quantity
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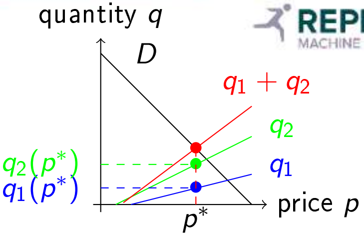


Definition (Pay-as-bid auction)

1. Agent set $\mathcal{N} = \{1, 2\}$
2. Action set:

$$\mathcal{A} := \{q : \mathbb{R}^+ \to \mathbb{R}\,,\ q \text{ non-decreasing function of price}\,,\ q \text{ invertible}\}$$

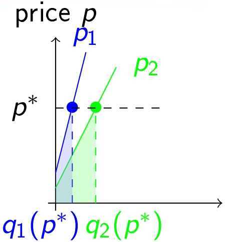
3. Utility of firm $i \in \mathcal{N}$:

$$u_i^{\mathrm{B}}(q_i, q_j) = \int_0^{q_i(p^*)} p_i^+(q)\,\mathrm{d}q - C(q_i(p^*))$$

where p_i is the inverse of q_i and p^* is the equilibrium price as before.

Model description: pay-as-bid auction

- 2 firms
- **Cost functions** $C(q)$, q quantity
- **Demand function** $D(p)$, p price.



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where p_i is the inverse of q_i and p^* is the equilibrium price as before.

Main results and current work

Assumptions: affine demand, quadratic costs.

1. Uniform price auction
   - ⋆ Linear supply functions: existence and uniqueness of NE.
   - ⋆ Affine supply functions: existence of an infinite number of NE.
2. Pay-as-bid auction
   - ⋆ Linear supply functions: existence and uniqueness of NE.
   - ⋆ Affine supply functions: non-existence of NE.

Current work

- Production capacity constraints.
- Two-stage game.
- Mechanism design.
- Evaluate model with real data (Italian electricity market).

Dataset: all submitted offers and bids of the Italian electricity market for each hour of each day from 2015 to 2018 with outcomes.

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Thank you for the attention

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